June 15th, 2010

# Math 251

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Summer 2010

Name: ........................................

Student ID (last 6 digits): XXX- .................

## Midterm 1

You have 60 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

**Show all work!** (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

**Good luck!**

| Question | Max. Points | Score |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 20 | |
| Total | 100 | |

**1)** *Quickies!* You don't need to justify your answers.

(a) If $A$ is square matrix and $\det A = 0$, what can you say about the number of solutions of $A\mathbf{x} = \mathbf{b}$?

(b) If $A$ is square matrix and for some vector $\mathbf{b}$ the system $A\mathbf{x} = \mathbf{b}$ has no solution, then what can you say about the number of solutions of $A\mathbf{x} = \mathbf{0}$? [Here, $\mathbf{0}$ is the zero vector.]

(c) If $A = \begin{bmatrix} 2 & \sqrt{3} & -\pi & 12/131 \\ 0 & -1 & \mathrm{e}^2 & \ln(10) \\ 0 & 0 & 7 & \cos(3) \\ 0 & 0 & 0 & 1 \end{bmatrix}$, then what are the $(4,1)$ and $(3,3)$ entries of $A^{-1}$?

(d) If $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions, then what can be said [for sure] about the reduced [row] echelon form of $A$?

(e) Write $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ as a product of elementary matrices.

**2)** Solve the systems $A\mathbf{x} = \mathbf{b}$ below. (These should be quick and you do not have to show work.)

(a) $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$.

(b) $A = \begin{bmatrix} 2 & 3 & -1 & 4 & 5 \\ 0 & 0 & 2 & -1 & 3 \\ 1 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 3 \end{bmatrix}$.

(c) $A = \begin{bmatrix} 1 & 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$.

**3)** Find the inverse of $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 3 & 1 & 5 & 6 \\ 0 & 2 & -1 & 8 \\ 2 & 0 & 4 & 3 \end{bmatrix}$.

**4)** Let $A = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & -1 \end{bmatrix}$. Compute $((A + 2B) \cdot C)^{\mathrm{T}}$.

**5)** Let $A = \begin{bmatrix} 0 & 1 & 7 & 1 & 0 \\ 2 & 5 & -1 & 3 & 0 \\ -1 & 2 & 1 & 5 & 1 \\ 3 & 1 & 3 & -1 & 0 \\ -1 & 1 & -3 & 2 & 0 \end{bmatrix}$.

(a) What is the cofactor of $A$ at position $(3, 3)$?

(b) If $B = [b_{i,j}]$ is the adjoint of $A$, then what is $b_{3,2}$? [Be careful here!]

**Scratch:**