

Math 251

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Name:

Student ID (last 6 digits): XXX-

MIDTERM 1

You have 60 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 6 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	15	
2	15	
3	15	
4	15	
5	15	
6	25	
Total	100	

1) [15 points] Solve the systems $A\mathbf{x} = \mathbf{b}$, given that [no need to show work on this one]:

$$(a) [A | \mathbf{b}] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & -1 \end{array} \right]$$

$$(b) [A | \mathbf{b}] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$(c) [A | \mathbf{b}] \sim \left[\begin{array}{ccccc|c} 1 & 0 & 2 & 0 & -1 & 4 \\ 0 & 1 & -1 & 0 & 2 & 2 \\ 0 & 0 & 0 & 1 & 1 & -1 \end{array} \right].$$

2) [15 points] If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{bmatrix}$, find $(AB)_{2,2}$ and $(AB)_{1,3}$. [**Remember:** $(C)_{i,j}$ is the entry of C in the i -th row and j -th column.]

3) [15 points] If A is an 3×3 matrix with $\det(A) = 2$, then what is $\det((2A^T)^{-1})$? [Show steps!]

4) [15 points] Let $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

(a) Find D^{-1} , or justify if D is not invertible.

(b) If B and C are square matrices of the same size, then complete the formula:

$$(B \cdot C)^{-1} =$$

(c) If $A^{-1} = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 0 & 3 & 0 \end{bmatrix}$, then, with the D above, find $(DA)^{-1}$. [If DA is not invertible, justify.]

5) [15 points] Let $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 5 & -1 & 1 \\ 1 & 2 & 0 & 1 \\ 2 & -1 & 1 & 0 \end{bmatrix}$. Given that $\det(A) = 15$, what is $(A^{-1})_{3,2}$?

[Hint: You do *not* need to compute the whole inverse to find this! With the right “tool”, you can compute it directly.]

6) [25 points] Let $A = \begin{bmatrix} 2 & 0 & 4 & 2 \\ 3 & 1 & 5 & 6 \\ 0 & 2 & -1 & 8 \\ 2 & 0 & 4 & 3 \end{bmatrix}$. Find the reduced row echelon form of A , $\det(A)$,

and, if possible, A^{-1} . [**Hint:** Don't work harder than you need to! You can compute all these *together!*]

Scratch: