Midterm (Take Home)

You must upload the solutions to this exam by 11:59pm on Tuesday 06/16. Since this is a take home, I want all your solutions to be neat and well written.
You can look at your notes, class discussions on SMC, *our* book, our videos and solutions posted by me, but you cannot look at any other references (including the Internet) and you cannot discuss this with *anyone*!
You can use a computer only to check your answers, as **you need to show work in all questions**.

1) [15 points] Use the *Extended Euclidean Algorithm* to write the GCD of 1183 and 826 as a linear combination of themselves. *Show the computations explicitly!* [**Hint:** You should get 7 for the GCD!]

2) [13 points] Compute the LCM of 1183 and 826 [the same numbers above!].

3) [15 points] Find the remainder of the division of 9482^{1532} when divided by 5 [i.e., what is 9482^{1532} congruent to modulo 5]. *Show your computations explicitly!*

4) [15 points] Give the set of all solutions of the system

$$\begin{aligned} 4x &\equiv 5 \pmod{15} \\ 5x &\equiv 22 \pmod{33} \end{aligned}$$

[**Hint:** The system *does* have solution(s)!]

5) [12 points] Suppose that

$$\begin{aligned} m &= 2^a \cdot 3^2 \cdot 5^b \cdot 7^3, \\ n &= 2^5 \cdot 3^c \cdot 5^4 \cdot 7^d, \\ \gcd(m,n) &= 2^5 \cdot 3^2 \cdot 5 \cdot 7^2, \\ \operatorname{lcm}(n,m) &= 2^7 \cdot 3^2 \cdot 5^4 \cdot 7^3. \end{aligned}$$

Find a, b, c and d.

6) [15 points] Let a, b and c be positive integers and suppose that there are $r, s, t \in \mathbb{Z}$ such that

$$ra + sb + tc = 1.$$

Prove that $\gcd(a, b, c) = 1$.

7) [15 points] Let p be a prime. Prove that for any integer a such that $p \nmid a$, the equation $x^p - x + a = 0$ never has an *integral* [i.e., in $\mathbb{Z}$] solution.
[**Hint:** As I've mentioned before, if an equation has an integral solution, it has a solution modulo any m.]