## Math 447: Homework $1^*$

## Due date: Wednesday September 3, 2014.

- 1. For each  $n \in \mathbb{N}$ , let  $A_n = \{(n-1)k : k \in \mathbb{N}\}.$ 
  - (a) Determine the truth value of the statement: for all  $n \in \mathbb{N}$ ,  $A_n \subseteq \mathbb{N}$ . Justify your answer.
  - (b) Find  $A_2 \cap A_3$ .
  - (c) Find  $\cup_{n\in\mathbb{N}}A_n$  and  $\cap_{n\in\mathbb{N}}A_n$ .
- 2. Suppose that  $A = B := \{x \in \mathbb{R} : 0 \le x \le 1\}$ . Consider the subset  $C := \{(x, y) : x = y^2 + 1\}$  of  $A \times B$ . Is set C a function? Justify your answer.
- 3. Suppose  $f : A \to B$  is a function.
  - (a) If E and F are subsets of A, prove that
    - i.  $f(E \cup F) = f(E) \cup f(F)$ ii.  $f(E \cap F) \subseteq f(E) \cap f(F)$
  - (b) If G and H are subsets of B, prove that

i. 
$$f^{-1}(G \cup H) = f^{-1}(G) \cup f^{-1}(H)$$
.  
ii.  $f^{-1}(G \cap H) = f^{-1}(G) \cap f^{-1}(H)$ 

- (c) Give an example to show that equality in 3(a)(ii) need not hold in general.
- (d) Give an example to show that it is NOT in general true that  $f(E \setminus F) \subseteq f(E) \setminus f(F)$ .
- 4. In Section 1.1, do Problems: #16, 19, 20, 21, 22, 24.
- 5. Use Mathematical Induction to prove the following:

(a) For all 
$$n \in \mathbb{N}$$
,  $1^3 + 2^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ 

(b) For all 
$$n \in \mathbb{N}$$
,  $\sum_{k=1}^{n} (-1)^{k+1} k^2 = (-1)^{n+1} \frac{n(n+1)}{2}$ .

(c) For all  $n \in \mathbb{N}$  and  $n \ge 4, 2^n < n!$ .

<sup>\*</sup>This homework covers Section 1.1 and 1.2. Please do as many exercise as you can in the textbook on these section. Please let me know if you have any question.