Math 447: Homework 2*

Due date: Wednesday September 10, 2014.

1. For any $a, b \in \mathbb{R}$, such that $a < b$, show that $a < \dfrac{a+b}{2} < b$.
2. Let that $a, b, \in \mathbb{R}$, and suppose that for every $\epsilon > 0$ we have $a \leq b + \epsilon$. Show that $a \leq b$.
3. Prove that for any $a, b \in \mathbb{R}$,
$$\left(\frac{a+b}{2}\right)^2 \leq \frac{a^2+b^2}{2}.$$
Show that equality holds if and only if $a = b$.
4. (a) Suppose that $0 < c < 1$. Show that $0 < c^2 < c < 1$. Show also that for all $n \in \mathbb{N}$, $c^n \leq c$.
   (b) Suppose that $c > 1$. Show that $c^2 > c > 1$. Show also that for all $n \in \mathbb{N}$, $c^n \leq c$
5. (a) Show that if $a \in \mathbb{R}$, then $|a| = \sqrt{a^2}$.
   (b) If $a < x < b$ and $a < y < b$, show that $|x - y| < b - a$.
6. Find all $x \in \mathbb{R}$ that satisfy the inequality $|x| + |x + 1| < 2$.
7. Do exercise numbers $16, 17, 18, 19$ in Section 2.2.

*This homework covers Section 2.1 and 2.2. Please do as many exercise as you can in the textbook on these section. Please let me know if you have any question.