Math 447: Quiz 3

Name: Solution

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1. Suppose that $A \subset B$, where A and B are nonempty bounded subsets of $\mathbb{R}$. Show that

$$\inf B \leq \inf A.$$

Proof. Let $b = \inf B$. It suffices to show that b is a lower bound of A. But this is obvious as b is a lower bound for B and therefore for $A \subset B$ as well. $\square$

2. Find the supremum and infimum of the set $\left\{ \dfrac{1}{2^n} : n \in \mathbb{N} \right\}$. Justify your answer.

Solution.

$$\inf \left\{ \frac{1}{2^n} : n \in \mathbb{N} \right\} = 0.$$

To show this: first 0 is a lower bound since, $0 \leq 1/2^n$ for all $n \in \mathbb{N}$. Also for any $\epsilon > 0$, $1/\epsilon > 0$ and by Archemedean Property, there exists a natural number m such that $1/\epsilon < m$. But $m \leq 2^m$, and so $1/\epsilon < 2^m$, implying that $1/2^m < \epsilon$. That is any $\epsilon > 0$ cannot be a lower bound. And therefore, 0 is the greatest of all the lower bounds.

$$\sup \left\{ \frac{1}{2^n} : n \in \mathbb{N} \right\} = \frac{1}{2}.$$

This follows from the fact that $2^n \geq 2$, for all $n \in \mathbb{N}$. $\square$