Math 447: Quiz 4 Solution

Name:____________

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Use the definition of limit to prove that

$$\lim_{n\to\infty} \frac{2n+1000}{n-2} = 2$$

We will show that for any $\epsilon > 0$, there exits $N_\epsilon \in \mathbb{N}$ such that $\left|\frac{2n+1000}{n-2} - 2\right| < \epsilon$.

Scratch work:

$$\left|\frac{2n+1000}{n-2} - 2\right| = \left|\frac{2n+1000-2(n-2)}{n-2}\right| = \left|\frac{1004}{n-2}\right| = \frac{1004}{n-2}, \quad \text{for } n \geq 3.$$

So now $\frac{1004}{n-2} < \epsilon \Leftrightarrow \frac{1004}{\epsilon} + 2 < n$.

Given $\epsilon > 0$, choose a natural number $N > \max\left\{\frac{1004}{\epsilon} + 2, 3\right\}$. Then for all $n \geq N$, we have

$$\left|\frac{2n+1000}{n-2} - 2\right| = \frac{1004}{n-2} \leq \frac{1004}{N-2} < \epsilon,$$

as we wanted to show.