Math 371 — Computing Project #3
Oct. 16, 2018, Due Nov. 1, 2018

The object of this project is to implement several methods for finding a root of a function, a polynomial in this case, and observe the convergence rates of each. Consider the polynomial $f(x) = p(x) = 600x^4 - 550x^3 + 200x^2 - 20x - 1$. It has a root $r = .23235296474991712$ and We would like to approximate it using 3 methods: Bisection, Newton and Secant.

1. For the bisection method, start with $a_0 = .1,\ b_0 = 1$. Stop the iteration for n such that $b_n - a_n \leq 2 \times 10^{-5}$. This will ensure that $|c_n - r| \leq 10^{-5}$. Your output should consist of a single table of 5 columns listing the values of n, a_n, b_n, c_n, $e_n = |c_n - r|$, respectively.

2. For Newton's method, start with $x_0 = .1$ and iterate until $|x_n - x_{n-1}| \leq 10^{-10}$. Your output should consist of a table with 5 columns listing the values of n, x_n, $|x_n - x_{n-1}|$, $e_n = |x_n - r|$, $ratio = e_n/e_{n-1}^2$, respectively. As the convergence is quadratic in this case, the ratios should converge to $\frac{f''(r)}{2f'(r)}$. Verify that this is the case.

3. For the secant method, start with $x_0 = .1,\ x_1 = .12$ and iterate until $|x_n - x_{n-1}| \leq 10^{-10}$. Your output should consist of a table with 6 columns listing the values of n, x_n, $|x_n - x_{n-1}|$, $e_n = |x_n - r|$, $ratio1 = e_n/(e_{n-1}e_{n-2})$, $ratio2 = e_n/e_{n-1}^{\alpha}$ where $\alpha = (1 + \sqrt{5})/2$. The object here is to observe the expected convergence rate of the secant method. In particular,

$$\lim_{n\to\infty} \frac{e_n}{e_{n-1}e_{n-2}} = \frac{f''(r)}{2f'(r)} \quad \text{and} \quad \frac{e_n}{|e_{n-1}|^{\alpha}} \approx \text{ constant for n large.}$$

For the Newton and the Secant methods you should observe that $|x_n - x_{n-1}|$ and e_{n-1} tend towards equality as n increases. This is true for sequences that converge at a superlinear or faster rate and validate the use of $|x_n - x_{n-1}| < TOL$ as a stopping criterion.

To evaluate p and p', you should use Horner's algorithm which should be included in your writeup. Of course you could write $p(x) = -1 + x(-20 + x(200 + x(-550 + x(600))))$ but this does not have generality, say if n is large.

Also, make your programs modular, i.e. have p and p' as subprograms callable from the main. This way, if you change the function, then you don't have to change the main program.