# MATH 471 COMPUTING PROJECT III

Nov. 9, 2018 due Nov. 20, 2018

We would like to approximate the function $f(x) = \dfrac{1}{1+25x^2}$ on the interval [-1,1] by several approximation techniques in order to gain experience in the implementation of such numerical methods and to compare their performance. In particular, we will illustrate the so-called Runge phenomenon whereby polynomials of high degree are shown to behave in an unstable manner.

Let $h = 1/5$. Given the 11 nodes $x_j = -1 + jh, \quad j = 0, \cdots, 10$, obtain approximations to $f$ using each of the following methods:

(i) Lagrange polynomial interpolation
(ii) Piecewise linear interpolation
(iii) Piecewise cubic Hermite interpolation
(iv) Cubic spline interpolation with Natural B.C.

**Remarks**

- Use Newton's interpolatory divided differences to generate the coefficients of the interpolatory polynomial in part (1)(i). You will need to do this once. Also, use Horner's algorithm to evaluate the interpolant at a given point $x$. This will be needed to generate the plot points and also to evaluate the error.
- For the interpolation procedures in parts (ii), (iii) and (iv) the global interpolant is composed of 10 local functions each corresponding to a particular interval $[x_j, x_{j+1}]$. In order to evaluate the interpolant at a given point $x$, you need to find the interval $[x_j, x_{j+1}]$ that contains it in order to find the piece of the interpolant that corresponds to it. The formula $j = [5 + 5x]$ should work except for $x = 1$, which can be handled as an exception.
- For the piecewise cubic Hermite interpolant, use the 4 canonical basis functions on $[0, 1]$ discussed in class. You have the formulas for these.
- To construct the cubic spline interpolant you have to solve a linear system with a tridiagonal matrix. You may use any package to do so. But it is preferable if you use a banded solver with the appropriate type of storage. In fact, you already wrote such a code for Project2.
- Your presentation (output) should include for each of the above 4 procedures, on a separate page
  - A graph of the function $f$ and its approximation $f_{app}$.
  - the "discrete" maximum error

$$E = \max_{1 \le i \le 250} |f(\zeta_i) - f_{app}(\zeta_i)|$$

  where $\zeta_i$'s are 250 equally spaced points in [-1,1]. $E$ is an approximation of the "true" maximum error $\max_{-1 \le x \le 1} |f(x) - f_{app}(x)|$