MATH 371 COMPUTING PROJECT 5

Nov. 30, 2018 due Dec. 7, 2018

The goals of this project are 1) to implement several methods for the approximation of an initial value problem for ODE's and 2) to experimentally determine the rate of convergence of each method.

We consider the problem

$$y' = \frac{y}{t} - \frac{y^2}{t^2}, \quad 1 < t \le 2, \quad y(1) = 1.$$

The exact solution is given by $y(t) = \frac{t}{1+\ln t}$.

The methods to be applied are:

(1) The second order Taylor series method

$$y_{n+1} = y_n + hf(t_n, y_n) + \frac{h^2}{2}\big(f_t(t_n, y_n) + f_y(t_n, y_n)f(t_n, y_n)\big), \quad n = 0, \ldots, N-1.$$

(2) The four-stage Runge-Kutta method (studied in class)

(3) The 2-step method

$$y_{n+1} - y_n = \frac{h}{2}\left[3f(t_n, y_n) - f(t_{n-1}, y_{n-1})\right], \quad n = 1, \ldots, N-1.$$

Note that for this method, in addition to $y_0 = 1$ you need to generate y_1 using the RK method.

We define the error of a method at time T ($T = 2$ here) by $E(T, h) = |y_N - y(T)|$. Note that it is also a function of h and it is expected to be $O(h^m)$ where m is the order of the method. It is also the rate of convergence of the error (or the method) as a function of h as $h \to 0$. To determine the convergence rate of a method, we proceed as follows: We write $E(T, h) = ch^m$. We wish to determine the rate of convergence m which can be done without knowledge of the constant c. For this, we do two (or more) runs with two different stepsizes h_1 and h_2. Then

$$E(T, h_1)/E(T, h_2) = ch_1^m/ch_2^m = (h_1/h_2)^m \to m = \frac{\ln\big(E(T, h_1)/E(T, h_2)\big)}{\ln\big(h_1/h_2\big)}$$

For each method you need to provide the following output table

$1/h$	y_N	E(2,h)	rate
25			—
50			
100			
200			
400			

Print the values of y_N as $1.d_1 \ldots d_{12}$ (format $\%.12f$ in C). For the errors and the rates use the format $\%.5e$.

Start early. It is a good idea to show me your output before the deadline to have a chance to make corrections.

$1/h$	y_N	E(2,h)	rate
25			—
50			
100			
200			
400			

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25			—
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