Lecture 1: Pattern formation in particle systems: from spherical shells to regular simplices

Flocking and swarming models which seek to explain pattern formation in mathematical biology often assume that organisms interact through a force which is attractive over large distances yet repulsive at short distances. Suppose this force is given as a difference of power laws and normalized so that its unique minimum occurs at unit separation. We detail a phase transition in the mildly repulsive range of exponents, which separates a region where the minimum energy configuration is uniquely attained by a uniform distribution of organisms over a spherical shell, from a region in which it is uniquely attained — apart from translations and rotations — by equidistributing the organisms over the vertices of a regular top-dimensional simplex (i.e. an equilateral triangle in two dimensions and regular tetrahedron in three).

We explore the sense in which such configurations are stable fixed points for the dynamics of the aggregation equation, also known as the 2-Wasserstein gradient flow of the associated interaction energy.

Based on work with Cameron Davies (University of Toronto) and Tongseok Lim (of Purdue's Krannert School of Management) [78][79] at http://www.math.toronto.edu/mccann/publications

Lecture 2: On the Monopolist's Problem Facing Consumers with Nonlinear Price Preferences

The principal-agent problem is an important paradigm in economic theory for studying the value of private information; the nonlinear pricing problem faced by a monopolist is a particular example. In this lecture, we identify structural conditions on the consumers' preferences and the monopolist's profit functions which guarantee either concavity or convexity of the monopolist's profit maximization. Uniqueness and stability of the solution are particular consequences of this concavity. Our conditions are similar to (but simpler than) criteria given by Trudinger and others for prescribed Jacobian equations to have smooth solutions. By allowing for different dimensions of agents and contracts, nonlinear dependence of agent preferences on prices, and of the monopolist's profits on agent identities, we improve on the literature in a number of ways. The same mathematics can also be adapted to the maximization of societal welfare by a regulated monopoly.

In the classical case of bilinear preferences, we introduce a new duality for certifying solutions, which leads to a free boundary formulation for the missing region in the square example of Rochet and Choné (1998).

Based on work in progress with Kelvin Shuangjian Zhang (University of Waterloo); see also [64] at http://www.math.toronto.edu/mccann/publications

Lecture 3: A nonsmooth approach to Einstein's theory of gravity

The theory of metric measure spaces with upper dimension and lower Ricci curvature bounds has provided an extremely fruitful approach to nonsmooth Riemannian geometry in recent years. In this talk we explore an analogous framework for nonsmooth Lorentzian geometry, based on entropic convexity conditions along timelike geodesics of probability measures on spacetime.

Based in part on work with Clemens Sämann (University of Vienna) [68][79] at http://www.math.toronto.edu/mccann/publications

Lecture 4: Asymptotics near extinction for nonlinear fast diffusion on a bounded domain

On a smooth bounded Euclidean domain, Sobolev-subcritical fast diffusion with vanishing boundary trace is known to lead to finite-time extinction, with a vanishing profile selected by the initial datum. In rescaled variables, we quantify the rate of convergence to this profile uniformly in relative error, showing the rate is either exponentially fast (with a rate constant predicted by the spectral gap), or algebraically slow (which is only possible in the presence of non-integrable zero modes). In the first case, the nonlinear dynamics are well-approximated by exponentially decaying eigenmodes up to at least twice the gap; this refines and confirms a 1980 conjecture of Berryman and Holland. We also improve on more recent results, by providing a new and simpler approach which is able to accommodate the presence of zero modes, such as those that occur when the vanishing profile fails to be isolated (and possibly belongs to a continuum of such profiles).

Based on work with Beomjun Choi (Postech) and Christian Seis (Münster) [80] at http://www.math.toronto.edu/mccann/publications

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