

**Math 519 - Applied Math Seminar/Game Theory - Spring 2022**

Tu, Th 2:50–4:05 in Ayres 111,

**Professor:** Tim Schulze, 224C Ayres Hall, 974-4162, tschulze@utk.edu

**Office Hours:** Tu 4:15-5:15, or by appointment

**Text:** *Game Theory* by Maschler, Solan and Zamir

**Course Prerequisites:** Ability to program (e.g. Matlab)

**Grading:** Your grade will be determined by some homework assignments and a final project.

**Course Description:** This course will serve as a general introduction to Game Theory and explore a few topics in more detail. The class will be accessible to both undergraduate and graduate students, with the only prerequisite being some programming ability (e.g. MatLab). Game theory is the analysis of interactive decision making. It generalizes decision theory, which can be viewed as a type of optimization, to the case of multiple decision makers with different goals. It can be applied to many areas, including economics, political science, biology, and, of course, strategic games. Basic topics will include: Extensive and Strategic form games, the minimax theorem, Nash equilibrium, two-player zero-sum games, and mixed strategies. As time permits, topics beyond this may include behavior strategies and Kuhn's theorem, the fictitious play algorithm, von Neumann's poker model, and some elementary machine learning algorithms. The latter will form the basis for the final project.

**Dates to note:**

March 14–18, Spring break—no class.

Th. April 14, Spring Recess—no class.

Tu. May 10, last day of class.

A list of papers and other resources discussed in class:

1. Kuhn, H. W. (1950). "Simplified Two-Person Poker". In Kuhn, H. W.; Tucker, A. W. (eds.). *Contributions to the Theory of Games*. Vol. 1. Princeton University Press. pp. 97–103.
2. von Neuman, J. and Morgenstern, O. (1944). "Theory of Games and Economic Behavior."
3. Chen, B. and Ankenman, J. (2006). "The mathematics of poker."
4. Gillies, Mayberry and von Neumann (1953).
5. J. Robinson, "Annals of Math." (1951).
6. J. Heinrich, M. Lanctot, D. Silver (2015?) "Fictitious Self-Play in Extensive-Form Games" conference proceedings.
7. C. Watkins and P. Dayan, "Q-Learning" *Machine Learning* **8** 1992.
8. J. Nash and L. Shapley, "A Simple 3-Person Poker Game," 1950.
9. A. Jafari, A. Greenwald, D. Gondek and G. Ercal, "On No-Regret Learning, Fictitious Play and Nash Equilibrium," 2001.

Here is a summary of the first part of the four part project.

Game 1:

Each player antes an amount  $a$ , forming a pot  $p=2a$ .

Each player receives an independently chosen, uniformly distributed random integer between 1 and  $N$ .

Player 1 checks or bets  $b$ .

If player 1 checks, there is a showdown for  $p$ .

If player 1 bets, player 2 may fold or call.

If player 2 folds, player 1 wins the pot  $p$ .

If player 2 calls there is a showdown for  $p+2b$ .

Method 1:

In fictitious play, each player chooses the best pure strategy response to the average play of their opponent over previous iterations of the game.

The basic assignment is to upload two graphs: the betting fraction for each card  $i$  as a function of  $i$ , and the calling fraction for each card  $i$  as a function of  $i$ .

Game 2: The same as game 1, but

If player 1 checks, player 2 may bet or check.

If player 2 also checks, there is a showdown for  $p$ .

If player 2 bets, player 1 may fold or call.

If player 1 folds, player 2 wins the pot  $p$ .

If player 1 calls, there is a showdown for  $p+2b$ .