Conceptus XIX (1985) Nr.47/S.83-89

EVALUATING SOCIAL WELFARE FUNCTIONALS: A REPLY TO NURMI

by Carl Wagner, Knoxville

Editorial Remark

In their book *Rational Consensus in Science and Society* (1981) Keith Lehrer and Carl Wagner propose a formal theory of rational consensus which is intended to illuminate not only questions of distributive justice and social welfare, but also issues such as consensual probability. This theory is criticized by Hannu Nurmi in his article on "Social Choice Theory and Democracy" published 1984 in *Mind, Language and Society* (Conceptus-Studien 2, ed. by O. Neumaier). In particular, Nurmi criticizes the method for attaining rational consensus proposed by Lehrer and Wagner for accomplishing only very few social choice procedures as far as choice-theoretic properties are concerned; e.g., their theory is inappropriate for democratic decisions, because it is applicable only to situations in which the participants are genuinely disinterested. In the following, Carl Wagner, who is responsible for the formal part of that book, first sketches the theory proposed by Lehrer and himself as well as some of its consequences. He then answers Nurmi's points, which he considers are based on three misunderstandings: 1) Nurmi invokes evaluative criteria of a kind other than that which Lehrer and he regard as appropriate for judging their theory. 2) Nurmi aggregates von Neumann-Morgenstern utilities by arithmetic averaging, whereas Lehrer and Wagner indicated clearly that such utilities were not reasonable candidates for arithmetic averaging. 3) Even if a theory considers only a small part of consensual methods as rational, the fault may not lie with the theory, but rather with the world, i.e., with the conditions under which consensus generally comes about (as is shown, e.g., by democratic decisions). In his rejoinder Nurmi agrees with Wagner concerning this last point. However, he rejects the other arguments, concluding that the burden of proof still rests with Lehrer and Wagner.

* * *

Vorbemerkung der Redaktion

In ihrem 1981 erschienenen Buch *Rational Consensus in Science and Society* schlagen Keith Lehrer und Carl Wagner eine formale Theorie des rationalen Konsenses vor, die nicht nur Probleme wie Verteilungsgerechtigkeit und soziales Wohlergehen erklären, sondern etwa auch das Problem der Konsenswahrscheinlichkeit adäquat behandeln soll. Diese Theorie wird von Hannu Nurmi u.a. in seiner Arbeit über "Social Choice Theory and Democracy" kritisiert, die 1984 in dem Band *Mind, Language and Society* (= Conceptus-Studien 2, hg. von O. Neumaier) erschienen ist. Nurmi bemängelt insbesondere, daß das von Lehrer und Wagner vorgeschlagene Verfahren zur Gewinnung eines rationalen Konsenses, soweit entscheidungstheoretische Merkmale betroffen sind, nur eine sehr kleine Gruppe sozialer Entscheidungsverfahren erfaßt; z.B. sei die Theorie inadäquat für demokratische Entscheidungen, da sie nur auf Situationen anwendbar sei, in denen die Beteiligten keinerlei persönliche Interessen verfolgen. Carl Wagner, der für den formalen Teil jenes Buches verantwortlich zeichnet, skizziert hier zu-

nächst nochmals die von Lehrer und ihm vorgeschlagene Theorie und deren Konsequenzen und geht dann auf einige Bedenken von Nurmi ein, die s.E. auf drei Mißverständnissen beruhen: 1) Nurmi bewertet die Theorie von Lehrer und Wagner nach Kriterien anderer Art, als diese selbst angelegt wissen möchten. 2) Nurmi errechnet das arithmetische Mittel von Gruppenentscheidungen mittels Nutzenrechnung à la von Neumann-Morgenstern, die von Lehrer und Wagner als für diesen Zweck unbrauchbar abgelehnt wird. 3) Selbst wenn eine Theorie nur einen kleinen Teil sozialer Entscheidungen als rational auszeichnet, spricht dies nicht unbedingt gegen die Theorie, sondern möglicherweise gegen die Bedingungen, unter denen Konsens sonst zustande kommt (wie z.B. bei demokratischen Entscheidungen). Zwar stimmt Nurmi in seiner Duplik diesbezüglich mit Wagner überein, doch weist er dessen andere Argumente zurück, so daß s.E. Lehrer und Wagner immer noch den Beweis für die Brauchbarkeit ihrer Theorie schuldig sind.

* * * * *

1. The Consensus Problem

In *Rational Consensus in Science and Society* (1981) Keith Lehrer and I examined the applications of various sorts of averaging functions to two important problems in group decision theory, namely, the problem of aggregating a sequence of subjective probability distributions into a single consensual distribution, and the problem of deriving a consensual ordering of some set of policy alternatives from a matrix of utilities assigned to these alternatives by a set of individuals.

In the former case we discovered that two simple and intuitively appealing axioms of aggregation constituted necessary and sufficient conditions for the use of *weighted arithmetic means* as a method for constructing consensual probabilities.[1] Based on this axiomatic analysis we strongly endorsed the use of this class of averaging functions in probability aggregation problems.

In the latter case we were fortunate to be able to draw upon some beautiful and deep results from the recent social choice literature. Here, as might have been expected, a much richer set of possibilities presented themselves. Axiomatic analysis furnished necessary and sufficient conditions for the use of weighted arithmetic means, weighted geometric means, Rawlsian maximin rules, pure utilitarianism, and various weighted combinations thereof (see, e.g., d'Aspremont/Gevers 1977, and Roberts 1980). Each of the axiomatic characterizations postulated an independence condition and a Pareto condition reminiscent of the similarly named conditions of classical social welfare theory.[2] Indeed, they differed only in the sort of invariance conditions

---

[1] Probability aggregation methods based on weighted arithmetics averaging are precisely the class of methods which (1) assign a consensual probability to each proposition purely as a function of the probabilities assigned by individuals to that proposition and (2) respect their agreement in assigning a proposition zero probability. See Lehrer/Wagner (1981: Theorems 6.4 and 6.7) and McConway (1981).

[2] The independence condition is, however, considerably weaker than Arrow's classical "independence of irrelevant alternatives". See Lehrer/Wagner (1981: 117).

which they postulated. Such invariance conditions correlate in an obvious way with interpersonal comparability assumptions and with assumptions about the nature of the scales on which individuals assess utilities. They limit thereby the kinds of numerical relations among utilities that may be taken into account, isolating meaningful welfare relations from arithmetic artifacts which may misleadingly appear to convey information.

To illustrate this important point, suppose that one had reason to believe that individual utility assignments were merely ordinal and that utilitiy levels were not comparable across individuals. In other words, suppose that any modification of any individual's utility assignments expressed equally well his or her welfare under various alternatives so long as the magnitudes of these modified utilities preserved, for each individual, the inequality relations of his or her original assignments. The appropriate invariance condition in this case is so strong that it precludes all but dictatorial methods for ordering the alternatives in question. In such a case utilities provide no information over and above individual preferences and we find ourselves back in the clutches of the Arrow Impossibility Theorem.

Indeed, even if individual utility assignments may be assumed to be cardinal, i.e., meaningful up to positive affine transformations, if both utility levels and utility differences fail to be comparable across individuals, the appropriate invariance condition is still strong enough to preclude all but dictatorial ordering of the alternatives (see Roberts 1980: Theorem 3). Thus, even if individuals assign utilities by the method of von Neumann and Morgenstern, a method well suited to individual decisionmaking, there is no improvement in the possibility of avoiding non-dictatorial outcomes in group decisionmaking.

On the other hand, given stronger measurability-comparability assumptions, the correspondingly weaker invariance conditions admit the possibility of a wide variety of methods for producing a consensual ordering. For example, weighted arithmetic means provide the precisely appropriate method of aggregating utilities when they may be assumed to be cardinal with differences comparable across individuals.[3] Weighted geometric means provide the appropriate method when individuals assess utilities on ratio scales, with ratios comparable across individuals (see Lehrer/Wagner 1981: Theorem 6.10). Rawlsian maximin-type rules are admissible in the case of ordinal utilities comparable as to level across individuals (see d'Aspremont/Gevers 1977: Theorem 7). And when utilities are cardinal and comparable as to both level and differences, a rich variety of aggregation methods sensitive to both equity and efficiency considerations may be implemented (see Roberts 1980: Theorem 4).

In view of the above results, Lehrer and I took care to phrase our normative prescriptions in the realm of social choice in a highly qualified way. We endorsed weighted arithmetic means for groups who could agree that their utility assignments had a rigidity appropriate to warrant it, and made the same prescription for geometric means, maximin rules, and so on (see Lehrer/Wagner 1981: 118-121). We did not presume to predict the outcomes of this prescription, believing it to be a matter for empirical investigation to study and develop utility scales with the appropriate properties. We did,

---

[3] The first axiomatic characterization of this kind appears in Blackwell/Girshick (1954: Theorem 4.3.1). Their results were refined by d'Aspremont/Gevers (1977: Theorem 3) and further generalized by Roberts (1980: Theorem 2).

however, in the nature of our illustrative examples, suggest that measurability-comparability conditions sufficient to implement non-dictatorial decision-making might more easily be established in the case of *expert judgments* from a disinterested standpoint about *utilities to society* or to some group enterprise as a whole. In addition we expressed a cautious optimism that such conditions might be established in limited contexts for more traditional social choice problems in which individuals report the *personal utilities* which they anticipate receiving under various alternatives.[4]

2. Nurmi's Critique

In two recent articles Nurmi (1984, 1985) attempts to evaluate our claim that *weighted arithmetic means* are, in appropriate circumstances, the rational way of aggregating utilities. While he appears to agree with us that expert judgments might sometimes be so aggregated, he is skeptical about the possibility of ever aggregating personal utilities in this way. His strategy is to evaluate weighted arithmetic averaging as a social choice mechanism by means of a series of properties which have been endorsed for the *classical social welfare problem*, as construed by Arrow, involving the aggregation of individual preferences into a consensual preferential ordering. On all accounts, including satisfaction of the so-called consistency property, the weak axiom of revealed preference, path independence, and certain Condorcet conditions, he finds weighted arithmetic averaging to be deficient.

It seems to me, first of all, that since Nurmi has invoked criteria originally designed to evaluate social welfare *functions* (mappings from preference profiles in consensual orderings) in order to evaluate social welfare *functionals* (mappings from utility matrices to consensual orderings), he owes us a justification for using such criteria. In the presence of certain measurability-comparability conditions, as indicated in the previous section, utility matrices do furnish a broader informational base than unelaborated preferences. Might this not justify a modified sensibility regarding the appropriate evaluation criteria? Moreover, Nurmi has chosen only to analyse weighted arithmetic means, although they occupied no *a priori* privileged position in our catlogue of social welfare functionals.

Both these observations may be illustrated with reference to the Condorcet criterion, which demands that an alternative be ranked first if it defeats all other alternatives in pairwise comparisons adjudicated by majority rule. Weighted arithmetic means violate the Condorcet criteria, and Nurmi holds this fact against them. He remarks that such a departure from majority rule might be justified, in the case of aggregating expert judgments, by the differential weights which the experts receive. But he misses the important point that such departures may be justified in both expert judgment and personal utility aggregation problems, even when equal weights are assigned. If, after all, utility differences are interpersonally comparable, then it is not unreasonable to take them into account. Thus, A's much greater preference for x over y (as measured by the differences in utilities which he assigns to

---

[4] Sen (1977) has emphasized the importance of distinguishing the aggregation of expert judgments from the aggregation of personal utilities, or what he calls "interests"

those alternatives) can reasonably defeat the more modest preferences of B and C for y over x, even if A, B and C receive equal weight. Crude combinatorial majoritarianism yields to a subtler majoritarianism based on depth of preference.

It is instructive to note that social welfare functionals based on geometric averaging and the maximin rule also violate the Condorcet criterion. Had Nurmi analyzed such alternatives to arithmetic averaging, he might have come to suspect that when utility differences, ratios, or levels are vouchsafed, it is not unreasonable that they should influence social choice. The greater the variety of reliable information we possess, the more subtle our social choice of mechanisms can be.

In his discussion of the weak axiom of revealed preference and path independence Nurmi aggregates von Neumann-Morgenstern utilities by arithmetic averaging and constructs thereby examples which violate these criteria. But we indicated clearly that von Neumann-Morgenstern utilities were not reasonable candidates for arithmetic averaging (see Lehrer/Wagner 1981: 119-120). Hence, his counterexamples are irrelevant, even assuming the reasonableness of these criteria.

As regards the so-called consistency criterion, which requires that an alternative ranked first by two disjoint groups be ranked first by the union of these two groups, I would draw a distinction between situations involving the aggregation of judgments and those involving the aggregation of personal utilities.

In the former case, which turns on expertise and the exchange of information, the group as a whole need clearly not reflect the sentiments of subcommittees which have not been apprised of all the relevant information. In the presence of additional experts, individuals may decide to shift the weights which they heretofore assigned their subcommittee colleagues, with a consequent reordering of alternatives.

In the latter case, one would expect in general that individuals would be assigned equal weights, reflecting the essentially democratic enterprise in which they are engaged. In such situations it is easy to prove that an alternative ranked first by subgroups will be similarly ranked by the entire group. When, in extreme situations, groups of voters depart from equal weighting due to a consensus that one or more individuals' utility assignments are suspect on grounds of dishonesty or neurotic exaggeration (see Lehrer/Wagner 1981: 48-49), respect for the conclusions of subgroups, where such behavior might fail to be detected, is no more warrantèd than in the case of the aggregation of expert judgments.

Finally, although the normative prescriptions which Lehrer and I offered for problems of social choice were qualified by measurability-comparability conditions which we did not presume to vouchsafe, one might ask, as Nurmi does, what the prospects are for meeting such conditions.

For the problem of aggregating expert judgments it seems to me that the prospects are very good. In many enterprises, after all, there is an agreed upon or imposed measure of utility which all experts understand in the same way. Corporations measure utilities in terms of monetary profits, medical consultants in terms of cure probabilities, and fire departments in terms of

response times. Indeed, the problem for such decisionmakers is not a lack of comparability across individual judgments, but rather a richness of comparability which allows consideration of a variety of concerns. The question is not whether there is a logical basis for considering such concerns but, rather, how to weigh them.

The problem of aggregating personal utilities is complicated by the idiosyncrasies of human nature and individual circumstance. As a result, the prospect of developing a comprehensive measure of personal utility with the requisite comparability properties is daunting. But it may be possible to develop interpersonally comparable measures of individual utility in limited contexts, though these will inevitably be qualified by the proviso *ceteris paribus*.

Steven Strasnick (1971) has, for example, analyzed the problem of distributing a scarce drug and concluded, under certain assumptions about the reactions of individuals to the disease which it is designed to alleviate, that immunity probabilities offer a utility measure with comparabilities requisite for implementing a utilitarian distributional scheme. While this example relies, among other things, on the expert appraisal of individual immunity probabilities and thus, deals with personal utilities highly influenced by expert judgment, it furnishes a prototype of the modest, context-sensitive analysis required to implement the theorems of axiomatic social choice theory. It also suggests that the implementation of such theorems may depend on *creating* comparability conditions, rather than simply *discovering* them.[5] Ruth Marcus (1980) has observed that when two ethical norms are incompatible, the fault may lie rather with the world as we have allowed it to persist than with the norms. Might the possibility of activating the theorems of social choice theory not supply an agenda for creating a society in which the premisses of these theorems have empirical content?

References

Blackwell, D. / Girshick, G.H. (1954): *Theory of games and statistical decisions*. New York.

d'Aspremont, C. / Gevers, L. (1977): Equity and the informational basis of collective choice. *Review of Economic Studies* 46: 199-210.

Lehrer, K. / Wagner, C. (1981): *Rational consensus in science and society*. Dordrecht-Boston/MA.

Marcus, R.B. (1980): Moral dilemmas and consistency. *Journal of Philosophy* 77: 121-136.

[5] Strasnick's example assumes a disease which causes comparable suffering to those who contract it, so that differences in immunity probabilities are comparable. Medical advances and a general improvement in individual health might make such an assumption roughly tenable. Similarly, monetarily denominated gains or losses might more plausibly be compared across individuals if the disparities in their net worth were not extreme.

McConway, K.J. (1981): Marginalization and linear opinion pools. *Journal of the American Statistical Association* 76: 410-414.

Nurmi, H. (1984): Social choice theory and democracy. In: Neumaier, O., ed. (1984): *Mind, language and society*. Vienna: 179-187.

Nurmi, H. (1985): Some properties of the Lehrer-Wagner method for reaching rational consensus. *Synthese* 62: 13-24.

Roberts, K. (1980): Interpersonal comparability and social choice theory. *Review of Economic Studies* 48: 421-439.

Sen, A. (1977): Social choice theory: A re-examination. *Econometrica* 45: 53-84.

Strasnick, S. (1979): Moral structures and axiomatic theory. *Theory and Decision* 11: 195-206.