Math 241 (1:50-2:40pm) Quiz #5 Fall 2024 Name:

1. Find the directional derivative $D_{\mathbf{u}}f(4,2,2)$ of $f(x,y,z) = \sqrt{x+y^2+z^3}$ in the direction $\mathbf{v} = \langle 1,2,2\rangle$. Find the direction that leads to the maximum of $D_{\mathbf{u}}f(4,2,2)$ and the value of $D_{\mathbf{u}}f(4,2,2)$ in that direction.

Solution.

$$\nabla f(x,y,z) = \frac{1}{2\sqrt{x+y^2+z^3}}\langle 1, 2y, 3z^2\rangle$$

and therefore $\nabla f(1,1,1) = \langle \frac{1}{8}, \frac{1}{2}, \frac{3}{2}\rangle$.

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \left\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right\rangle$$

Thus,

$$D_{\mathbf{u}}f(1,1,1) = \nabla f(1,1,1)\cdot \mathbf{u} = \frac{1}{8}\times\frac{1}{3}+\frac{1}{2}\times\frac{2}{3}+\frac{3}{2}\times\frac{2}{3} = \frac{33}{24}$$

The direction leading to the maximal rate is $\nabla f(1,1,1) = \langle \frac{1}{8}, \frac{1}{2}, \frac{3}{2}\rangle$. The maximal rate is

$$\|\nabla f(1,1,1)\| = \sqrt{\left(\frac{1}{8}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{3}{2}\right)^2} = \sqrt{\frac{161}{64}}$$

2. Find the equation of the tangent plane to the surface $x^2-2x+y^2-4y+z^2-6z+11 = 0$ at the point $(2,3,3)$.

Solution. Set $F(x,y,z) = x^2-2x+y^2-4y+z^2-6z+11$. Then $\nabla F(x,y,z) = \langle 2x-2, 2y-4, 2z-6\rangle$ and the normal vector $\mathbf{n} = \nabla F(2,3,3) = \langle 2,2,0\rangle$. The equation of the tangent plane is

$$2(x-2)+2(y-3) = 0 \quad \text{or} \quad x+y=5$$