Quiz #6 (13:50-2:40pm) **Fall 2024** **Name:**

1. Find the critical points of the function $f(x, y) = x^2 - y^2 - 2x - 4y$ and then classify them into local maximum, local minimum and saddle points.

Solution. $f_x = 2x - 2$ and $f_y = -2y - 4$. So the critical point is $(1, -2)$. $f_{xx} = 2$, $f_{yy} = -2$ and $f_{xy} = 0$. Therefore, $D(x, y) = f_{xx} f_{yy} - [f_{xy}]^2 = -4$. Notice $D(1, 2) = -4 < 0$. By the second derivative test, $(1, 2)$ is a saddle point.

2. Find the point on the plane $x - y + z = 4$ that is closest to the point $(1, 2, 3)$.

Solution. We need to minimize the square of the distance $(x-1)^2 + (y-2)^2 + (z-3)^2$. Notice that $z = 4 - x + y$. So we need to minimize

$$f(x, y) = (x - 1)^2 + (y - 2)^2 + (1 - x + y)^2$$

Notice that $f_x = 2(x - 1) - 2(1 - x + y) = 4x - 2y$ and that $f_y = 2(y - 2) + 2(1 - x + y) = -2x + 4y - 2$. Solve the equations $4x - 2y = 0$ and $-2x + 4y - 2 = 0$ gives the critical point is $(1/3, 2/3)$. and therefore $z = 4 - (1/3) + (2/3) = 13/3$. **Conclusion:** $(1/3, 2/3, 13/3)$ is the point on the plane that is closest to $(1, 2, 3)$.