Math 241(1:50-2:40pm) **Quiz #7** **Fall 2024** **Name:**

Evaluate the double integrals

1. $\iint_D ydA$, where the domain D is bounded by the curves $y = \sqrt{x}$ and $y = x^2$.

Solution 1. (Treated as type-1 doamain)

$$\iint_D ydA = \int_0^1 \left[\int_{x^2}^{\sqrt{x}} ydy\right]dx = \frac{1}{2}\int_0^1 y^2\Big|_{y=x^2}^{y=\sqrt{x}} dx = \frac{1}{2}\int_0^1 (x - x^4)dx = \frac{1}{2}\Big(\frac{1}{2} - \frac{1}{5}\Big) = \frac{3}{20}$$

Solution 2. (Treated as type-2 doamain)

$$\iint_D ydA = \int_0^1 \left[\int_{y^2}^{\sqrt{y}} ydx\right]dy = \int_0^1 y(\sqrt{y} - y^2)dy = \int_0^1 (y^{3/2} - y^3)dy = \frac{2}{5} - \frac{1}{4} = \frac{3}{20}$$

2. $\iint_D e^{-x^2-y^2} dA$ where $D = \{(x, y)|\ x^2 + y^2 \leq 1,\ y \geq 0\}$ is a simi-disc.

Solution. Under the polar coordinates, the domain D be comes $\{(r, \theta)|\ 0 \leq \theta \leq \pi$ and $0 \leq r \leq 1\}$. Therefore,

$$\iint_D e^{-x^2-y^2} dA = \int_0^\pi \int_0^1 e^{-r^2} rdrd\theta = \pi \int_0^1 e^{-r^2} rdr$$

Under the substitution $u = r^2$, $du = 2rdr$.

$$\int_0^1 e^{r^2} rdr = \frac{1}{2}\int_0^1 e^{-u}du = \frac{1}{2}(1 - e^{-1})$$

In summary

$$\iint_D e^{-x^2-y^2} dA = \frac{\pi}{2}(1 - e^{-1})$$