Math 241(1:50-2:40pm)) **Quiz #8** **Fall 2024** **Name:**

1. Find the area of the part of the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the plane $z = 0$.

Solution. $\dfrac{\partial z}{\partial x} = -2x$ and $\dfrac{\partial z}{\partial y} = -2y$. Let $D = \{(x, y)|x^2 + y^2 \leq 1\}$.

$$S = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} dA = \iint_D \sqrt{1 + 4x^2 + 4y^2} dA$$
$$= \int_0^{2\pi} \int_0^1 \sqrt{1 + 4r^2} r dr d\theta = 2\pi \int_0^1 \sqrt{1 + 4r^2} r dr = \frac{\pi}{4} \int_1^5 \sqrt{u} du = \frac{\pi}{6}(5^{3/2} - 1)$$

2. Evaluate the tripple integral $\iiint_E z dV$, where $E = \{(x, y, z)|\ x \geq 0,\ y \geq 0 \text{ and } 0 \leq z \leq 1 - x^2 - y^2\}$

Solution. Let $D = \{(x, y)|x^2 + y^2 \leq 1,\ x \geq 0,\ y \geq 0\}$. By Fubini's theorem

$$\iiint_E x dV = \iint_D \left[\int_0^{1-x^2-y^2} z dz\right] dA = \frac{1}{2} \iint_D (1 - x^2 - y^2)^2 dA$$
$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} \int_0^1 (1 - r^2) r dr d\theta = \frac{\pi}{4} \int_0^1 (1 - r^2)^2 r dr \overset{u=1-r^2}{=} \frac{\pi}{8} \int_0^1 u^2 du = \frac{\pi}{24}$$