Math 241(1:50-2:40pm) **Quiz #9** **Fall 2024** **Name:**

Evaluate the following triple integrals:

1. $\iiint_E z^2\sqrt{x^2+y^2}dV$ where $E=\{(x,y,z)|x^2+y^2\le 1 \text{ and } 0\le z\le 2\}$.

Solution. By the cylindrical substitution

$$\iiint_E z^2\sqrt{x^2+y^2}dV = \int_0^{2\pi}\int_0^1\int_0^2 z^2 rrdzdrd\theta$$
$$= 2\pi\left(\int_0^1 r^2dr\right)\left(\int_0^2 z^2dz\right) = 2\pi\times\frac{1}{3}\times\frac{2^3}{3} = \frac{16\pi}{9}$$

2. $\iiint_E \sqrt{x^2+y^2+z^2}dV$, where $E=\{(x,y,z)|1\le x^2+y^2+z^2\le 4 \text{ and } z\ge 0\}$.

Solution. By the sphereical substitution

$$\iiint_E \sqrt{x^2+y^2+z^2}dV = \int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_1^2 \rho\cdot\rho^2\sin\varphi d\rho d\varphi d\theta$$
$$= 2\pi\left(\int_0^{\frac{\pi}{2}}\sin\varphi d\varphi\right)\left(\int_1^2\rho^3d\rho\right) = \frac{15\pi}{2}$$