Math 241 **Quiz #3 (12:40-1:30pm)** **Fall 2024** **Name:**

1. Find a parametric equation of the tangent line to the curve $\mathbf{r} = \langle \cos 2t, \sin t, t\rangle$ at $t = \pi/4$.

Solution. When $t = \pi/4$, $x = \cos(\pi/2) = 0$, $y = \sin(\pi/4) = \sqrt{2}/2$ and $z = \pi/4$. That gives the information of "one point". In addition, $\mathbf{r}'(t) = \langle -2\sin 2t, \cos t, 1\rangle$. This gives the information about "one direction":

$$\mathbf{v} = \mathbf{r}'\left(\frac{\pi}{4}\right) = \langle -2\sin\frac{\pi}{2}, \cos\frac{\pi}{4}, 1\rangle = \langle -2, \frac{\sqrt{2}}{2}, 1\rangle$$

So the parametric equation of the tangent line is

$$\begin{cases} x = 0 - 2t = -2t \\ y = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}t \\ z = \frac{\pi}{4} + t \end{cases}$$

2. Find the length of the curve $\mathbf{r}(t) = \langle t, 3\cos t, 3\sin t\rangle$ over $0 \le t \le 1$.

Solution. $\mathbf{r}'(t) = \rangle 1, -3\sin t, 3\cos t\rangle$ and

$$\|\mathbf{r}'(t)\| = \sqrt{1^2 + 9\sin^2 t + 9\cos^2 t} = \sqrt{10}$$

$$s = \int_0^1 \|\mathbf{r}'(t)\| dt = \int_0^1 \sqrt{10}dt = \sqrt{10}$$