Math 241 (12:40-1:30pm) Quiz #4 Fall 2024 Name:

1. Find an equation of the tangent plane to the surface given by $z = x/y^2$ at the point $(-4, 2, -1)$

Solution.

$$\frac{\partial z}{\partial x} = \frac{1}{y^2} \quad \text{and} \quad \frac{\partial z}{\partial y} = -2\frac{x}{y^3}$$

Therefore

$$\left.\frac{\partial z}{\partial x}\right|_{P_0} = \frac{1}{4} \quad \text{and} \quad \left.\frac{\partial z}{\partial y}\right|_{P_0} = 1$$

The equation of the tangent plane:

$$z + 1 = \frac{1}{4}(x + 4) + (y - 2) = \frac{1}{4}x + y - 1$$

2. Use the chain rule to find $\partial z/\partial s$ and $\partial z/\partial t$, given that $z = \ln(3x + 2y)$, $x = s\sin t$, $y = t\cos s$.

Solution.

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = \frac{3}{3x + 2y}\sin t - \frac{2}{3x + 2y}t\sin s$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = \frac{3}{3x + 2y}s\cos t + \frac{2}{3x + 2y}\cos t$$