Quiz #6 (12:40-1:30pm) **Fall 2024** **Name:**

1. Find the critical points of the function $f(x, y) = 4x + 6y - x^2 - y^2$ and then classify them into local maximum, local minimum and saddle points.

Solution. $f_x = 4 - 2x$ and $f_y = 6 - 2y$. So the critical point is $(2, 3)$. $f_{xx} = -2$, $f_{yy} = -2$ and $f_{xy} = 0$. Therefore, $D(x, y) = f_{xx}f_{yy} - [f_{xy}]^2 = 4$. Notice $D(2, 3) = 4 > 0$ and $f_{xx}(2, 3) = -2 < 0$. By the second derivative test, $(2, 3)$ is a local maximum point.

2. Find the point on the plane $x + y - z = 1$ that is closest to the point $(2, 1, -1)$.

Solution. We need to minimize the square of the distance $(x-2)^2+(y-1)^2+(z+1)^2$. Notice that $z = x + y - 1$. So we need to minimize

$$f(x, y) = (x - 2)^2 + (y - 1)^2 + (x + y)^2$$

Notice that $f_x = 2(x - 2) + 2(x + y) = 4x + 2y - 4$ and that $f_y = 2(y - 1) + 2(x + y) = 2x + 4y - 2$. Solving $4x + 2y - 4 = 0$ and $2x + 4y - 2 = 0$ yields $x = 1$ and $y = 0$ and therefore $z = 1 + 0 - 1 = 0$. **Conclusion:** $(1, 0, 0)$ is the point on the plane that is closest to $(2, 1, -1)$.