Math 241 (12:40-1:30pm) Quiz #7 Fall 2024 Name:

Evaluate the double integrals

1. $\iint_D xdA$, where the domain D is bounded by the curves $y = x$ and $y = x^2$.

Solution 1. (Treated as type-1 doamain)

$$\iint_D xdA = \int_0^1 \left[\int_{x^2}^{x} xdy \right] dx = \int_0^1 x(x - x^2)dx = \int_0^1 (x^2 - x^3)dx = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

Solution 2. (Treated as type-2 doamain)

$$\iint_D xdA = \int_0^1 \left[\int_y^{\sqrt{y}} xdx \right] dy = \frac{1}{2} \int_0^1 x^2 \Big|_y^{\sqrt{y}} dy = \frac{1}{2} \int_0^1 (y - y^2)dy = \frac{1}{2}\Big(\frac{1}{2} - \frac{1}{3}\Big) = \frac{1}{12}$$

2. $\iint_D (x^2 + y^2)dA$, where $D = \{(x, y) |\ x^2 + y^2 \leq 1 \text{ and } x \geq 0\}$

Solution. Under the polar coordinates, the domain D be comes $\{(r, \theta) |\ \frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2} \text{ and } 0 \leq r \leq 1\}$. Therefore,

$$\iint_D (x^2 + y^2)dA = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^1 r^2 rdrd\theta = \pi \int_0^1 r^3 rdr = \frac{\pi}{4}$$