Math 241(12:40-1:30pm) **Quiz #8** **Fall 2024** **Name:**

1. Find the area of the part of the sphere $z = x^2 - y^2$ that lies within the cylinder $x^2 + y^2 = 1$.

Solution. $\dfrac{\partial z}{\partial x} = 2x$ and $\dfrac{\partial z}{\partial y} = -2y$. Let $D = \{(x, y) | x^2 + y^2 \leq 1\}$.

$$S = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} dA = \iint_D \sqrt{1 + 4x^2 + 4y^2} dA$$
$$= \int_0^{2\pi} \int_0^1 \sqrt{1 + 4r^2} r dr d\theta = 2\pi \int_0^1 \sqrt{1 + 4r^2} r dr = \frac{\pi}{4} \int_1^5 \sqrt{u} du = \frac{\pi}{6}(5^{3/2} - 1)$$

2. Evaluate the tripple integral $\iiint_E x dV$, where E is bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$

Solution. Let $D = \{(x, y) | x + y \leq 1,\ x, y \geq 0\}$. By Fubini theorem

$$\iiint_E x dV = \iint_D \left[\int_0^{1-x-y)} x dz\right] dA = \iint_D x\left[1 - x - y\right] dA$$
$$= \int_0^1 x\left[\int_0^{1-x} \left[1 - x - y\right] dy\right] dx = \int_0^1 x\left[(1 - x)^2 - \frac{1}{2} y^2\Big|_{y=0}^{y=1-x}\right] dx$$
$$= \frac{1}{2} \int_0^1 x(1 - x)^2 dx = \frac{1}{2} \int_0^1 \left[x - 2x^2 + x^3\right] dx = \frac{1}{2}\left[\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right] == \frac{1}{24}$$