Math 241(12:40-1:30pm) **Quiz #9** **Fall 2024** **Name:**

Evaluate the following triple integrals:

1. $\iiint_E z\sqrt{x^2+y^2}dV$ where $E = \{(x,y,z)|1 \le x^2+y^2 \le 4 \text{ and } 0 \le z \le 3\}$.

Solution. By the cylindrical substitution

$$\iiint_E z\sqrt{x^2+y^2}dV = \int_0^{2\pi}\int_1^2\int_0^3 zrrdzdrd\theta$$
$$= 2\pi\Big(\int_1^2 r^2dr\Big)\Big(\int_0^3 zdz\Big) = 2\pi \times \frac{2^3-1}{3} \times \frac{9}{2} = 21\pi$$

2. $\iiint_E \sqrt{x^2+y^2+z^2}dV$, where $E = \{(x,y,z)|x^2+y^2+z^2 \le 1 \text{ and } z \ge 0\}$.

Solution. By the sphereical substitution

$$\iiint_E \sqrt{x^2+y^2+z^2}dV = \int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^1 \rho\cdot\rho^2\sin\varphi d\rho d\varphi d\theta$$
$$= 2\pi\Big(\int_0^{\frac{\pi}{2}} \sin\varphi d\varphi\Big)\Big(\int_0^1 \rho^3 d\rho\Big) = \frac{\pi}{2}$$