Math 241 Quiz #1 Fall 2024 (1:50-2:40pm) Name:

1. Find the angle between the vectors $\mathbf{a} = \langle 1, 1, 1\rangle$ and $\mathbf{b} = \langle 1, -2, 1\rangle$.
Solution.

$$\mathbf{a} \cdot \mathbf{b} = 1 \times 1 + 1 \times (-2) + 1 \times 1 = 0$$

$\mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other. Therefore The angle $\theta = \frac{\pi}{2}$

2. Given three points $P(1, 4, 6)$, $Q(-2, 0, -1)$ and $R(1, -1, 1)$ in space, find the area of the triangle ΔPQR and a vector that perpendicular to the triangle.

Solution. $\overrightarrow{PQ} = \langle -3, -4, -7\rangle$ and $\overrightarrow{PR} = \langle 0, -5, -5\rangle$.

$$\overrightarrow{PQ} \times \overrightarrow{PR} == \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -4 & -7 \\ 0 & -5 & -5 \end{vmatrix} = \mathbf{i}\begin{vmatrix} -4 & -7 \\ -5 & -5 \end{vmatrix} - \mathbf{j}\begin{vmatrix} -3 & -7 \\ 0 & -5 \end{vmatrix} + \mathbf{k}\begin{vmatrix} -3 & -4 \\ 0 & -5 \end{vmatrix}$$
$$= -15\mathbf{i} - 15\mathbf{j} + 15\mathbf{k}$$

This is a vector perpendicular to the triangle. The area of the triangle is

$$\frac{1}{2}\|\overrightarrow{PQ} \times \overrightarrow{PR}\| = \frac{1}{2}\sqrt{15^2 + 15^2 + 15^2} = \frac{5}{2}\sqrt{8^2 + 3^2 + 3^2} = \frac{5}{2}\sqrt{3}$$