Math 241 (1:50am-2:40pm) Quiz #10 Fall 2024 Name:

Given the field $\mathbf{F} = \langle y^2 - 2x, 2xy + y^3 \rangle$,

(1). determine wether or not $\mathbf{F}$ is conservative (To receive credit for your answer, you have to give the reason).

Solution.

$$\frac{\partial F_1}{\partial y} = 2y \quad \text{and} \quad \frac{\partial F_2}{\partial x} = 2y$$

They are equal. So $\mathbf{F}$ is conservative.

(2). find a function $f(x, y)$ such that $\mathbf{F} = \nabla f$.

Solution. First, $\frac{\partial f}{\partial x} = y^2 - 2x$ and $\frac{\partial f}{\partial y} = 2xy + y^3$. From the first equation, $f(x, y) = xy^2 - x^2 + C(y)$. Taking the partial derivative: $\frac{\partial f}{\partial y} = 2xy + C'(y)$. Comparing it to the second equation, $c'(y) = y^3$. Thus, $C(y) = \frac{1}{4}y^4$. Finally,

$$f(x, y) = xy^2 - x^2 + \frac{1}{4}y^4$$

(3). Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is a curve on the plane going from $(0, 0)$ to $(1, 1)$.

Solution. By fundamental theorem,

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \nabla f \cdot d\mathbf{r} = f(1, 1) - f(0, 0) = 1 - 1 + \frac{1}{4} = \frac{1}{4}$$