

1. Solve the equation $\frac{dx}{dt} = 3xt^2$.

Solution.

$$\frac{dx}{x} = 3t^2 dt, \quad \ln x = t^3 + \ln C, \quad x = Ce^{t^3} \quad \text{or} \quad x = Ce^{t^3}$$

2. 2. Solve the initial value problem $\frac{dy}{dx} = x(1 + y^2)$, $y(0) = 0$.

Solution.

$$\frac{dy}{1 + y^2} = x dx, \quad \tan^{-1} y = \frac{1}{2}x^2 + C$$

When $x = 0$, $0 = \tan^{-1}(0) = C$. So $\tan^{-1} y = \frac{1}{2}x^2$. The final answer is $y = \tan \frac{x^2}{2}$