

Find the general solution to $y'' - 3y' + 2y = e^t$

Solution. The general solution is $y = \tilde{y} + y_p$. Solve $r^2 - 3r + 2 = 0$ get $r_1 = 1$ and $r_2 = 2$. So the general solution to the homogeneous equation $y'' - 3y' + 2y = 0$ is

$$\tilde{y} = C_1 e^t + C_2 e^{2t}$$

Since e^t is a solution to the homogeneous equation. So we set up $y_p = Ate^t$. Then $y_p' = Ae^t(1+t)$ and $y_p'' = Ae^t(t+2)$.

$$y_p'' - 3y_p' + 2y_p = Ae^t((t+2) - 3(t+1) + 2t) = -Ae^t$$

Solve $-Ae^t = e^t$ we have $A = -1$. Hence, $y_p = -te^t$. The general solution is

$$y = C_1 e^t + C_2 e^{2t} - te^t$$