Math 241 **Quiz #5** **Spring 2025** **Name:**

Use Lagrange multipliers to find the minimal value of the function $f(x,y,z) = x^2 + y^2 + z^2$ subject to the constraint $2x + 4y + 4z = 1$.

Solution. $f(x,y,z) = x^2 + y^2 + z^2$ and $g(x,y,z) = 2x + 4y + 4z$.

$$\nabla f(x,y,z) = \langle 2x, 2y, 2z \rangle, \qquad \nabla g(x,y,z) = \langle 2, 4, 4 \rangle$$

We solve

$$\begin{cases} \nabla f(x,y,z) = \lambda \nabla g(x,y,z) \\ \\ 2x + 4y + 4z = 1 \end{cases} \quad \text{or} \quad \begin{cases} 2x = 2\lambda \\ 2y = 4\lambda \\ 2z = 4\lambda \\ 2x + 4y + 4z = 1 \end{cases}$$

$$\begin{cases} x = \lambda \\ y = 2\lambda \\ z = 2\lambda \\ 2\lambda + 8\lambda + 8\lambda = 1 \end{cases} \quad \text{or} \quad \begin{cases} x = \lambda \\ y = 2\lambda \\ z = 2\lambda \\ \lambda = \frac{1}{18} \end{cases}$$

Hence, $x = 1/18$, $y = 2/18$, $z = 2/18$

$$f\Big(\frac{1}{18}, \frac{2}{18}, \frac{2}{18}\Big) = \Big(\frac{1}{18}\Big)^2 + \Big(\frac{2}{18}\Big)^2 + \Big(\frac{2}{18}\Big)^2 = \frac{9}{18^2} = \frac{1}{36}$$

So the minimal value is $1/36$.