Math 231 **Quiz #9** **Spring 2026** **Name:**

1. Find the inverse Laplace transform

$$\mathcal{L}^{-1}\Big(\frac{1}{s(s^2+1)}e^{-3s}\Big)$$

Solution.

$$\mathcal{L}^{-1}\Big(\frac{1}{s(s^2+1)}e^{-3s}\Big) = \mathcal{L}^{-1}\Big(\frac{1}{s(s^2+1)}\Big)(t-3)\cdot u(t-3)$$

By the decomposition

$$\frac{1}{s(s^2+1} = \frac{1}{s} - \frac{s}{s^2+1}$$

$$\mathcal{L}^{-1}\Big(\frac{1}{s(s^2+1)}\Big) = \mathcal{L}^{-1}\Big(\frac{1}{s}\Big) - \mathcal{L}^{-1}\Big(\frac{s}{s^2+1}\Big) = 1-\cos t$$

Finally,

$$\mathcal{L}^{-1}\Big(\frac{1}{s(s^2+1)}e^{-3s}\Big) = \Big(1-\cos(t-3)\Big)\cdot u(t-3)$$

2. Solve the initial value problem $y'' + y = u(t-3)$, $y(0) = y'(0) = 0$ using the method of Laplace transform.

Solution. Taking Laplace transform

$$\mathcal{L}(y'') + \mathcal{L}(y) = \mathcal{L}(u(t-3)) = \frac{1}{s}e^{-3s}$$

$$s^2\mathcal{L}(y) + \mathcal{L}(y) = \frac{1}{s}e^{-3s}$$

Solving for $\mathcal{L}(y)$:

$$\mathcal{L}(y) = \frac{1}{s(s^2+1)}e^{-3s}$$

Hence,

$$y = \mathcal{L}^{-1}\Big(\frac{1}{s(s^2+1)}e^{-3s}\Big) = \Big(1-\cos(t-3)\Big)\cdot u(t-3)$$