hfill Math 231 **Quiz #2** **Spring 2026** **Name:**

1. Solve $\frac{dy}{dx} - \frac{y}{x} = xe^x$

Solution.

$$\int P(x)dx = -\int \frac{1}{x}dx = -\ln x$$

$$y = \exp\{\ln x\}\left[C + \int e^{-\ln x} xe^x dx\right] = x\left[C + \int e^x dx\right] = x(C + e^x)$$

2. Solve $(3x^2y + e^y)dx + (x^3 + xe^y - 2y)dy = 0$.

Solution.

$$\frac{\partial M}{\partial y} = 3x^2 + e^y = \frac{\partial N}{\partial x}$$

The equation is exact. Solve

$$\begin{cases} \dfrac{\partial F}{\partial x} = 3x^2y + e^y \\ \dfrac{\partial F}{\partial y} = x^3 + xe^y - 2y \end{cases}$$

From the first equation, $F = x^3y + xe^y + C(y)$. Therefore $\frac{\partial F}{\partial y} = x^3 + e^y + C'(y)$. Comparing it to the second equation, $C'(y) = -2y$. Thus, $C(y) - y^2$. Therefore, $F(x, y) = x^3y + xe^y - y^2$. The solution is

$$x^3y + xe^y - y^2 = C$$