Math 241 **Quiz #1** **Spring 2025** **Name:**

1. Find the angle θ between the vectors $\mathbf{a} = \langle 1, 2, 2\rangle$ and $\mathbf{b} = \langle 1, 1, 0\rangle$.

Solution.

$$\mathbf{a}\cdot\mathbf{b} = 1\times 1 + 2\times 1 + 2\times 0 = 3$$

$$\|\mathbf{a}\| = \sqrt{1^2+2^2+2^2} = 3 \quad\text{and}\quad \|\mathbf{b}\| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$$

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Therefore $\theta = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$.

2. Given three points $P(1,0,0)$, $Q(1,1,0)$ and $R(1,1,1)$ in space, find the area of the triangle ΔPQR and a vector $\mathbf{n}$ that perpendicular to the triangle.

Solution. $\overrightarrow{PQ} = \langle 0,1,0\rangle$ and $\overrightarrow{PR} = \langle 0,1,1\rangle$.

$$\mathbf{n} = \overrightarrow{PQ}\times\overrightarrow{PR} == \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \mathbf{i}\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - \mathbf{j}\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} + \mathbf{k}\begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} = \mathbf{i}$$

The area of the triangle is

$$\frac{1}{2}\|\mathbf{n}\| = \frac{1}{2}\|\mathbf{i}\| = \frac{1}{2}$$