Math 241 **Quiz #2** **Spring 2025** **Name:**

1.Find an equation of the plane through the points $(0,1,1)$, $(1,0,1)$ and $(1,1,0)$.

Solution. Name these three points as P, Q, R, respectively. $\overrightarrow{PQ} = \langle 1,-1,0\rangle$ and $\overrightarrow{PR} = \langle 1,0,-1\rangle$.

$$\overrightarrow{PQ} \times \overrightarrow{PR} == \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \mathbf{i}\begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} - \mathbf{j}\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} + \mathbf{k}\begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix}$$
$$= \mathbf{i} + \mathbf{j} + \mathbf{k}$$

The equation of the plane:

$$(x-0)+(y-1)+(z-1)=0 \quad \text{or} \quad x+y+z=2$$

2. Find a parametric equation of the tangent line to the curve $\mathbf{r} = \langle \cos 2t, \sin t, t\rangle$ at $t = \pi/4$.

Solution. When $t=\pi/4$, $x=\cos(\pi/2)=0$, $y=\sin(\pi/4)=\sqrt{2}/2$ and $z=\pi/4$. That gives the information of “one point”. In addition, $\mathbf{r}'(t) = \langle -2\sin 2t, \cos t, 1\rangle$. This gives the information about “one direction”:

$$\mathbf{v} = \mathbf{r}'\left(\frac{\pi}{4}\right) = \langle -2\sin\frac{\pi}{2}, \cos\frac{\pi}{4}, 1\rangle = \langle -2, \frac{\sqrt{2}}{2}, 1\rangle$$

So the parametric equation of the tangent line is

$$\begin{cases} x = 0 - 2t = -2t \\ y = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}t \\ z = \frac{\pi}{4} + t \end{cases}$$