Math 241 **Quiz #6** **Spring 2025** **Name:**

Evaluate the double integrals

1. $\iint_D x dA$, where the domain D is bounded by the curves $y = x$ and $y = \sqrt{x}$.

Solution 1. (Treated as type-1 doamain)

$$\iint_D x dA = \int_0^1 \left[\int_x^{\sqrt{x}} x dy\right] dx = \int_0^1 x(\sqrt{x} - x) dx = \int_0^1 (x^{3/2} - x^2) dx = \frac{2}{5} - \frac{1}{3} = \frac{1}{15}$$

Solution 2. (Treated as type-2 doamain)

$$\iint_D x dA = \int_0^1 \left[\int_{y^2}^{y} x dx\right] dy = \frac{1}{2}\int_0^1 x^2\Big|_{y^2}^{y} dy = \frac{1}{2}\int_0^1 (y^2 - y^4) dy = \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{1}{15}$$

2. $\iint_D (x^2 + y^2) dA$, where $D = \{(x, y) |\ 1 \leq x^2 + y^2 \leq 4\}$

Solution. Under the polar coordinates, the domain D be comes $\{(r, \theta) |\ 0 \leq \theta \leq 2\pi$ and $1 \leq r \leq 2\}$. Therefore,

$$\iint_D (x^2 + y^2) dA = \int_0^{2\pi} \int_1^2 r^2 r dr d\theta = 2\pi \int_1^2 r^3 dr = \frac{15\pi}{2}$$