Math 241 **Quiz #8** **Spring 2025** **Name:**

Find the work done by the force field $\mathbf{F}(x, y, z) = \langle 2xy,\ x^2 + 2yz,\ y^2 \rangle$ in moving a particle from $(0, 0, 0)$ to $(1, 1, 1)$ along the following two different paths

(1). The path $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ $(0 \le t \le 1)$.

Solution.

$$W = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_C 2xydx + (x^2 + 2yz)dy + y^2dz$$
$$= \int_0^1 \Big[2t \cdot t^2 + (t^2 + 2t^2 \cdot t^3)2t + t^4 \cdot 3t^2\Big]dt$$
$$= \int_0^1 \Big[4t^3 + 7t^6\Big]dt = 1 + 1 = 2$$

(2). The **straight** path from $(0, 0, 0)$ to $(1, 1, 1)$

Solution. The parametric equation for the line is $x = t$, $y = t$, $z = t$ $(0 \le t \le 1)$

$$W = \int_C 2xydx + (x^2 + 2yz)dy + y^2dz = \int_0^1 \Big[2t \cdot t + (t^2 + 2t \cdot t) + t^2\Big]dt$$
$$= 6\int_0^1 t^2dt = 2$$