

Evaluate the following triple integrals:

1. $\iiint_E z\sqrt{x^2+y^2}dV$ where $E = \{(x, y, z) | 1 \leq x^2 + y^2 \leq 4 \text{ and } 0 \leq z \leq 3\}$.

Solution. By the cylindrical substitution

$$\begin{aligned}\iiint_E z\sqrt{x^2+y^2}dV &= \int_0^{2\pi} \int_1^2 \int_0^3 z r r dz dr d\theta \\ &= 2\pi \left(\int_1^2 r^2 dr \right) \left(\int_0^3 z dz \right) = 2\pi \times \frac{2^3 - 1}{3} \times \frac{9}{2} = 21\pi\end{aligned}$$

2. $\iiint_E \sqrt{x^2+y^2+z^2}dV$, where $E = \{(x, y, z) | x^2 + y^2 + z^2 \leq 1 \text{ and } z \geq 0\}$.

Solution. By the sphereical substitution

$$\begin{aligned}\iiint_E \sqrt{x^2+y^2+z^2}dV &= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^1 \rho \cdot \rho^2 \sin \varphi d\rho d\varphi d\theta \\ &= 2\pi \left(\int_0^{\frac{\pi}{2}} \sin \varphi d\varphi \right) \left(\int_0^1 \rho^3 d\rho \right) = \frac{\pi}{2}\end{aligned}$$